Math 106 — Quiz 1

Instructions. The quiz should take 10 minutes. Show brief justification for all answers.

Problem. Consider a continuous-time Markov chain $(X_t)_{t\geq 0}$ on the state space $\Omega = \{1, 2, 3, 4\}$. with generator (rate matrix)

$$Q = \begin{pmatrix} -1 & 0 & 0 & 1 \\ 0 & -5 & 4 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 1 & -1 \end{pmatrix}.$$

(a) Draw the directed graph associated with this generator. Each edge should be labeled by the rate associated with that transition. Is this process reducible?

(b) Find $\lim_{t\to\infty} \mathbb{E}[X_t^2 | X_0 = 1]$

(c) If the chain starts in state 2, what is the probability that the first jump is to state 4?