Probability Models (Evans and Rosenthal 1.2)

Sample space (S): set of all possible outcomes of experiment / simulation / data collection

Events (E): all subsets of outcomes

(Def 1.2.1)
Probability Model: assigns #s in $[0,1]$ to events

$$P: E \to [0,1]$$

Example (Flipping a fair coin)

assumptions: only land on two sides
initial conditions don't matter
fair

$S = \{H, T\}$

$E = \{\emptyset, \{H\}, \{T\}, \{H, T\}\}$

$P(\emptyset) = 0$

$P(\{H\}) = P(\{T\}) = \frac{1}{2}$

$P(\{H, T\}) = 1$

Notation
often write
$P(\{H\}) = P(H)$
for single outcome in S

Basic Properties of Probability Models

1) $P(A) \in [0,1] \quad \forall A \in E$

2) $P(\emptyset) = 0$

3) $P(S) = 1$

4) If $A_1, A_2, A_3 \ldots$ are disjoint

then

$$P(A_1 \cup A_2 \cup A_3 \ldots) = \sum_{i=1} P(A_i)$$

E.g. $P(H) + P(T) = 1$

Example (flipping a unfair coin)

$$P(H) = q > \tfrac{1}{2} \Rightarrow P(T) = 1 - q$$

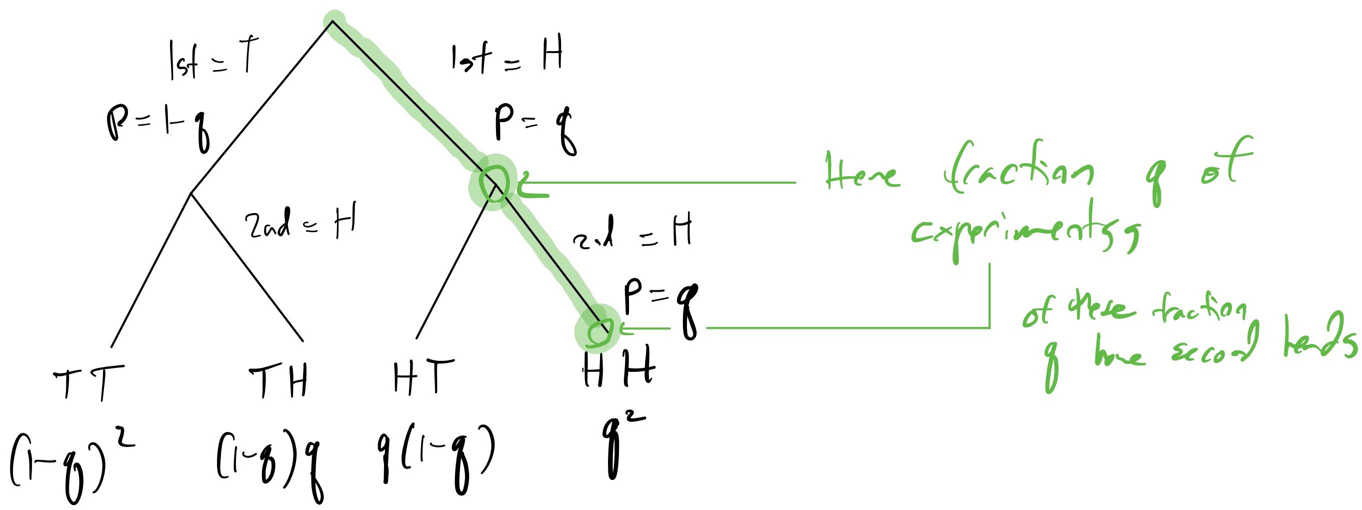
Long run frequency interpretation

$$P(H) = \frac{\# \text{ of heads}}{\# \text{ of flips}}$$ when # of flips is very large

Key idea: use freq interpretation to calculate more complex probabilities

Flip coin twice

$$S = \{(H,H), (H,T), (T,H), (T,T)\}$$



$$(1-q)^2 + 2(1-q)q + q^2 = 1 - \cancel{2q} + \cancel{q^2} + \cancel{2q} - \cancel{2q^2} + \cancel{q^2}$$

$$= 1$$

$$P\left(\begin{matrix}\text{exactly one}\\ \text{heads}\end{matrix}\right) = 2(1-q)q$$

<u>See also</u> Examples 1.2.5, 1.2.5, 1.2.6

Random Variable (different definition than 2.1.1)

A random variable is a variable whose value is determined by chance via a probability model

Usually we prefer to work w/ numerical variables

e.g. $H \to 1$, $T \to 0$

Bernoulli Random variable

Our model of a coin is called a Bernoulli Random variable

we write

$$X \sim \text{Bernoulli}(\theta)$$

to mean that X is a Bernoulli random variable w/ parameter θ

$$\to \quad P(X=0) = 1-\theta$$
$$P(X=1) = \theta$$

In general, we write

$$\underline{X} \sim \text{Name of Distribution (Parameters)}$$

Probability distribution $\equiv$ probability model